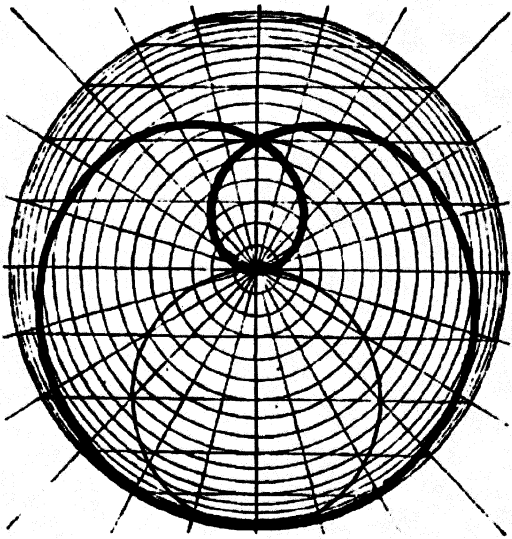
GEOMETRIC MAPS

EXHIBITING THE METHOD OF DELINEATING CURVES THROUGH THE INTERSECTIONS OF TRIGONOMETRIC LINES.

Fig. 1.

vel $\left\{\begin{matrix}1:1,\text{ dir.}\\1:2,\text{ inv.}\end{matrix}\right\}$ 2nd. BICIRCLOID. $\left\{\begin{matrix}x=\cos\phi+e\cos 2\phi;\\y=\sin\phi+e\sin 2\phi.\end{matrix}\right.$

$r^2=1+e^2+2e\cos\phi$. and $r^4-(1+2e^2)r^2-e^2ap=2ex$.



Rad. = 1 : 1; $\therefore e=1$. centric.

The TRISECTRIX. $r=a\cos\frac{\theta}{3}$.

$x=\cos\phi+\cos 2\phi$.

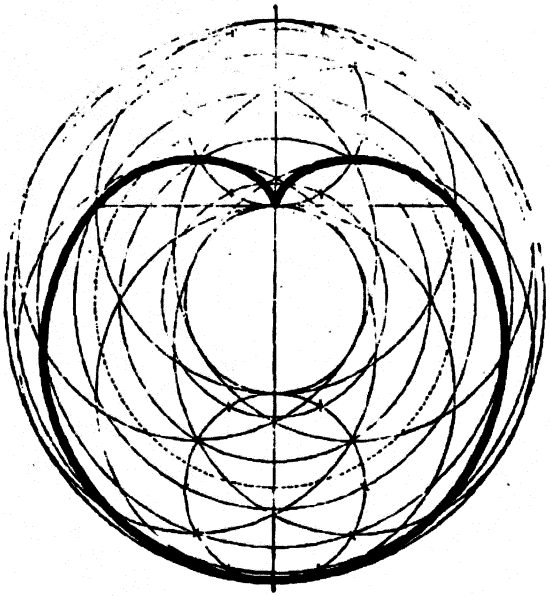
$y=\sin\phi+\sin 2\phi$.

$4r^4-3a^2r^2=a^2x=(x^2+y^2)\cdot(4x^2+4y^2-3a^2)$.

Fig. 2.

vel $\left\{\begin{matrix}1:1,\text{ dir.}\\1:2,\text{ inv.}\end{matrix}\right\}$ 2nd. **BICIRCLOID.** $\left\{\begin{matrix}x=\cos\phi+e\cos 2\phi;\\y=\sin\phi+e\sin 2\phi.\end{matrix}\right.$

$r^2=1+e^2+2e\cos\phi$. and $r^4-(1+2e^2)r^2-e^2\,\mathfrak{a}\,\mathfrak{p}=2ex$.



Rad. = 2 : 1; $\therefore e=\frac{1}{2}$. cuspidate.

The CARDIOID. $r=D(1-\cos\theta)=2D\sin^2\frac{\theta}{2}$. (Origin at the Cusp.)

θ measured from pericenter.

$x=2\cos\phi-\cos 2\phi$;

$y=2\sin\phi-\sin 2\phi$.

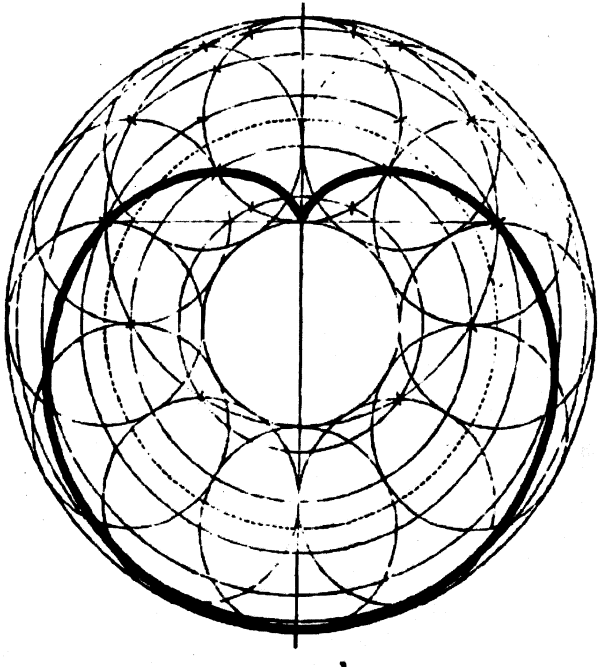
$\therefore r^4-\frac{3}{2}D^2r^2=\frac{3}{16}D^4-D^2x$; or $(r^2-\frac{3}{4}D^2)^2=\frac{3}{4}D^4-D^2x$.

(Origin at the Center.)

Fig. 3.

vel $\left\{\begin{matrix}1:1,\text{ dir.}\\1:2,\text{ inv.}\end{matrix}\right\}$ 2*nd.* **BICIRCLOID.** $\left\{\begin{matrix}x=\cos\phi+e\cos 2\phi;\\y=\sin\phi+e\sin 2\phi.\end{matrix}\right.$

$r^2=1+e^2+2e\cos\phi$. and $r^4-(1+2e^2)r^2-e^2\,\text{a}\,p=2ex$.



Rad. $=1:2$; $\therefore e=\frac{1}{2}$. cuspidate.

The CARDIOID. $r=2\text{D}\cos^2\frac{\theta}{2}=\text{D}(1+\cos\theta)$. (Origin at the Cu

θ measured from apocenter.

$x=2\cos\phi+\cos 2\phi;$

$y=2\sin\phi+\sin 2\phi.$

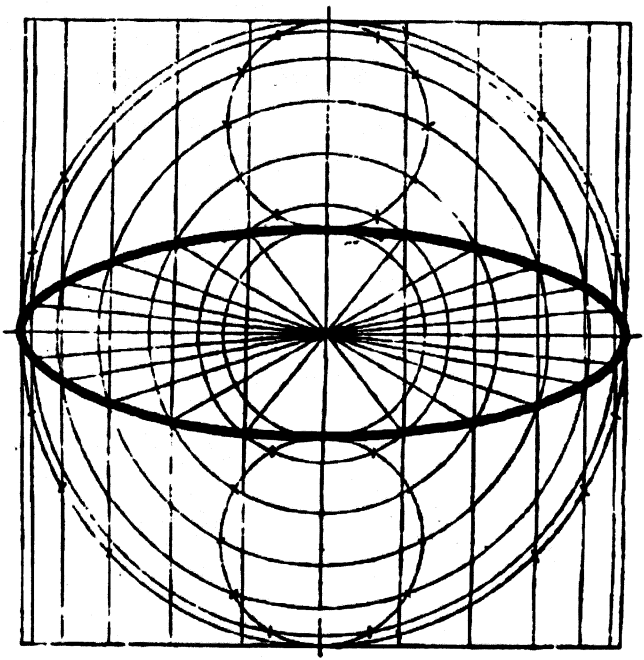
$\therefore r^4-\frac{3}{2}\text{D}^2r^2=\frac{3}{16}\text{D}^4+\text{D}^3x$; or $(r^2-\frac{3}{4}\text{D}^2)^2=\frac{3}{4}\text{D}^4+\text{D}^3x$.

(Origin at the Center.)

Fig. 4.

vel $\left\{\begin{matrix}2:1,\\ \text{inv.}\end{matrix}\right\}$ 3rd. BICIRCLOID. $\begin{cases}x=\cos\phi+e\cos\phi=a\cos\phi;\\ y=\sin\phi-e\sin\phi=p\sin\phi.\end{cases}$

$r^2=1+e^2+2e\cos 2\phi$. The Ellipse. $a^2y^2+p^2x^2=a^2p^2$.



Rad. $=1:2$; $\therefore e=\frac{1}{2}$. transcentric.

An Ellipse.

$x=2\cos\phi+\cos\phi=3\cos\phi$;

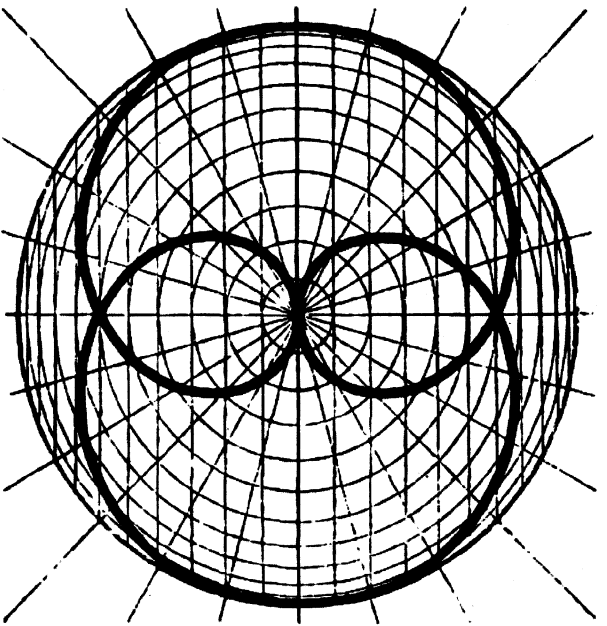
$y=2\sin\phi-\sin\phi=\sin\phi$.

$9y^2+x^2=9=r^2+8y^2$.

Fig. 5.

vel $\left\{\begin{matrix}2 : 1, \text{dir.}\\ 2 : 3, \text{inv.}\end{matrix}\right\}$ 4th. BICIRCLOID. $\left\{\begin{matrix}x = \cos\phi + e\cos 3\phi;\\ y = \sin\phi + e\sin 3\phi.\end{matrix}\right.$

$r^2 = 1 + e^2 + 2e\cos 2\phi$. and $(r^2 - y^2).(r^2 - a\,e)^2 = 4\,e\,x^2$.



Rad. = 1 : 1 ; $\therefore e = 1$. centric.

TWIN of the ELLIPSE. $r = a\cos\frac{\theta}{2}$.

Axis of x passes through the apocenter.

$x = \cos\phi + \cos 3\phi$;

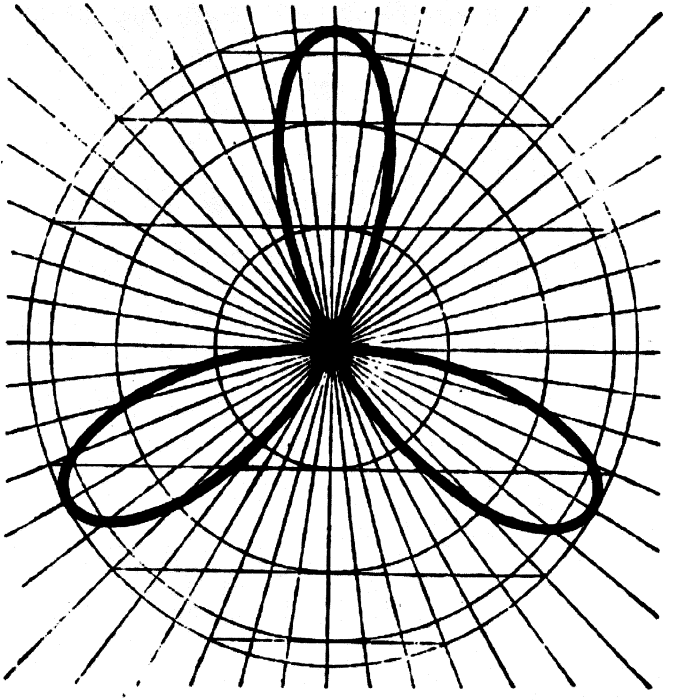
$y = \sin\phi + \sin 3\phi$.

$(x^2 + y^2).\{2(x^2 + y^2) - a^2\}^2 = a^4 x^2$.

Fig. 6.

vel $\left\{\begin{matrix}3 : 1, \text{inv.}\\ 3 : 2, \text{inv.}\end{matrix}\right\}$ 6th. **BICIRCLOID.** $\left\{\begin{matrix}x=\cos\phi+e\cos 2\phi;\\ y=\sin\phi-e\sin 2\phi.\end{matrix}\right.$

$r^2=1+e^2+2e\cos 3\phi.$ $e^2r^4+(1+2e^2)ar^2-a^2y^2=8ex^3-6exr^2.$



Rad.$=1:1$; $\therefore e=1$. centric.

$r=a\cos 3\theta.$

Axis of x passes through the apocenter.

$x=\cos\phi+\cos 2\phi;$

$y=\sin\phi-\sin 2\phi.$

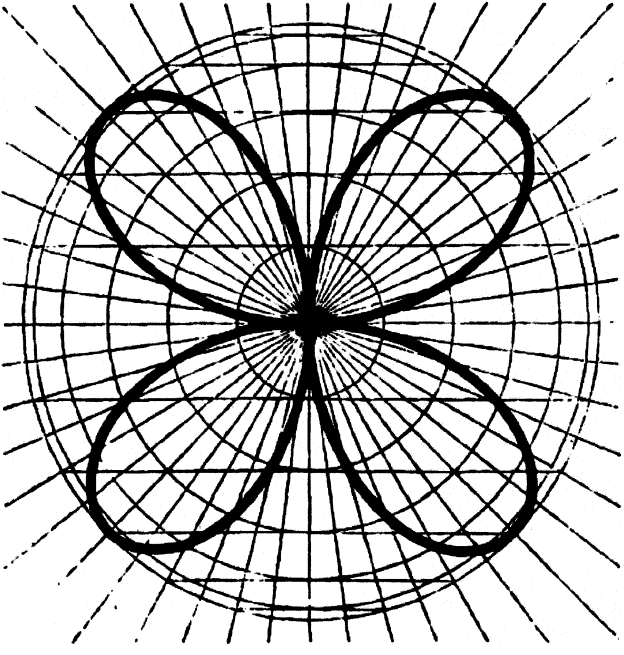
$(x^2+y^2)^2=ax\,(x^2-3y^2).$

Fig. 7.

·vel $\left\{\begin{matrix}4 : 1, \text{inv.}\\ 4 : 3, \text{inv.}\end{matrix}\right\}$ 11th. BICIRCLOID. $\left\{\begin{matrix}x=\cos\phi+e\cos 3\phi;\\ y=\sin\phi-e\sin 3\phi.\end{matrix}\right.$

$r^2=1+e^2+2e\cos 4\phi.$ $\quad 2er^4(\cos 4\theta-1)=-16ex^2y^2=$

$e^2r^8-(2e-e^3+3e^4)r^4+(1+3e^2)a^2p^2r^2-a^4p^6.$



Rad.$=1 : 1$; $\therefore e=1$. centric.

$r=a\sin 2\theta;\left\{\begin{matrix}x=\cos\phi-\cos 3\phi;\\ y=\sin\phi+\sin 3\phi.\end{matrix}\right\}\therefore (x^2+y^2)^3=4a^2x^2y^2.$

or

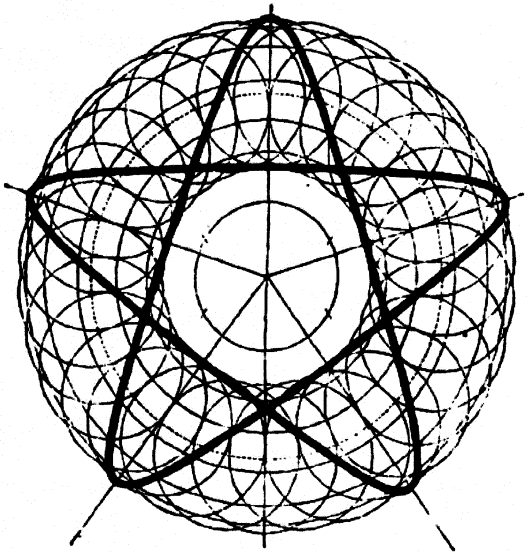
$r=a\cos 2\theta;\left\{\begin{matrix}x=\cos\phi+\cos 3\phi;\\ y=\sin\phi-\sin 3\phi.\end{matrix}\right\}\therefore (x^2+y^2)^3=a^2(x^2-y^2)^2.$

According as the axis of x passes through the pericenter, or through the apocenter.

Fig. 8.

vel $\left\{\begin{matrix}5 : 2, \text{inv.}\\ 5 : 3, \text{inv.}\end{matrix}\right\}$ 19th. BICIRCLOID. $\left\{\begin{matrix}x=\cos 2\phi+e\cos 3\phi;\\ y=\sin 2\phi-e\sin 3\phi.\end{matrix}\right.$

$r^2=1+e^2+2e\cos 5\phi$. $2e^5r^5\cos 5\theta=32e^5x^5-40e^5r^2x^3+10e^5r^4x=e^5r^5+(1+e+3e^2)ap r^4-(2+3e^2)a^2p^2r^2+a^3p^3$.



Rad.$=4:9$. $\therefore e=\frac{4}{9}$. orthoidal.

PARABOLITE.

$x=9\cos 2\phi+4\cos 3\phi$;

$y=9\sin 2\phi-4\sin 3\phi$.

$2\times 9^5\times 4^5r^5\cos 5\theta=4^5r^5+(9^2+2\times 9^2\times 4^2-3\times 4^5)r^4+(3\times 4^5-7\times 9^2\times 4^3+3\times 9^4\times 4^1+3\times 9^2\times 4^3-2\times 9^5)r^3+13^5\times 5^5$:

or $23328\,r^5\cos 5\theta=373248\,x^5-366560\,r^2x^3+116640\,r^4x=256\,r^5+560625\,r^5-57671250\,r^3+1160290625$.

HENRY PERIGAL, Jun., Clayné Walk, Chelsea,—24th February, 1853.